Constraining The Distance-Redshift Relation Through Strong Gravitational Lensing of the CMB

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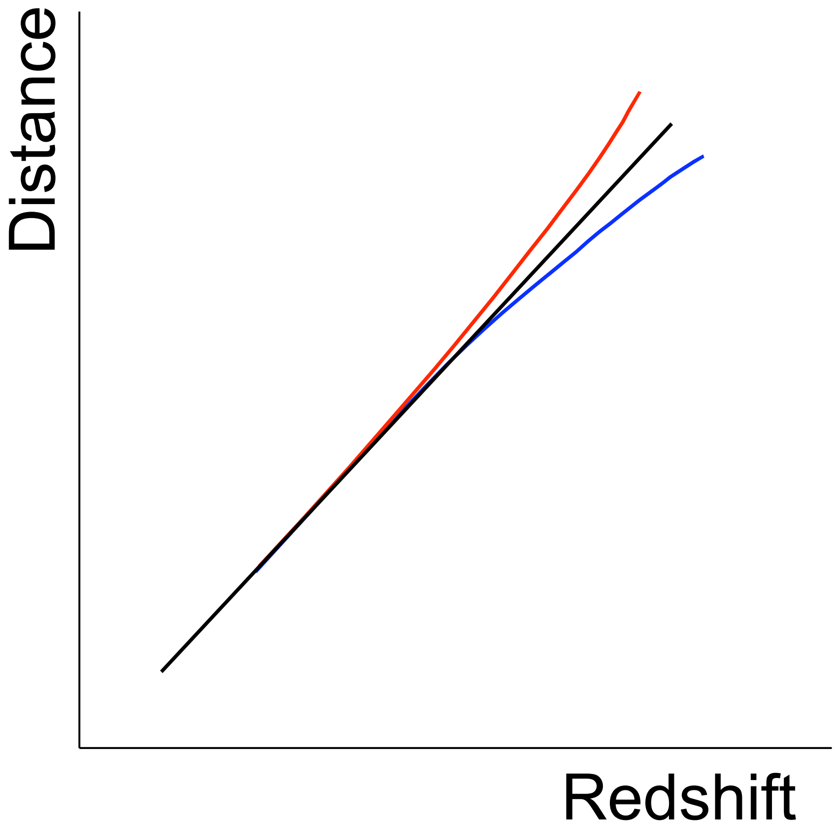
You all know...



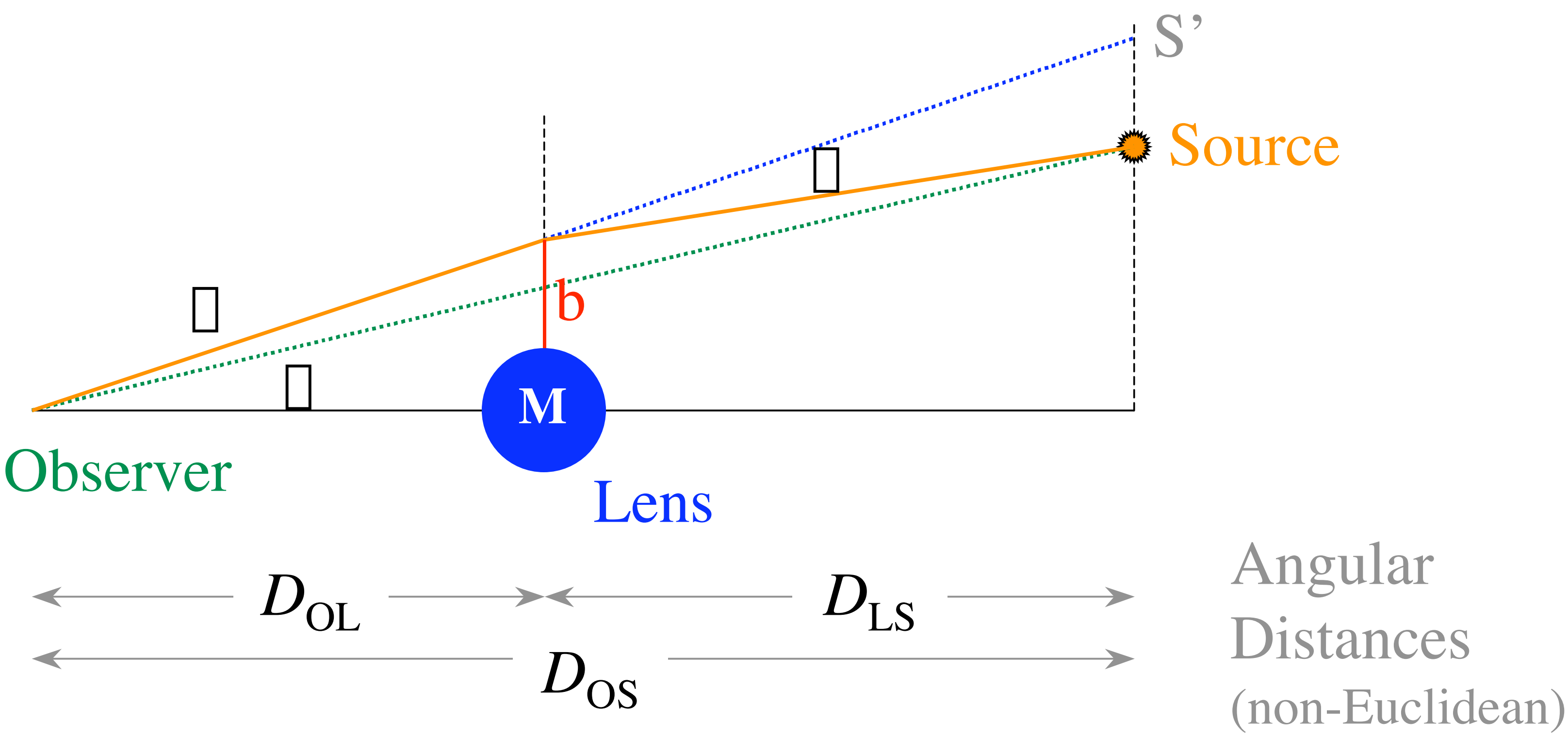
Robertson - Walker metric (flat space)

$$ds^2 = -dt^2 + a^2(t)\left[dx^2 + dy^2 + dz^2\right]$$

$$\left[a(t)\right]^{-1} = 1 + z(t)$$

Measuring $z(t)$, or z(distance), reveals expansion history of the Universe

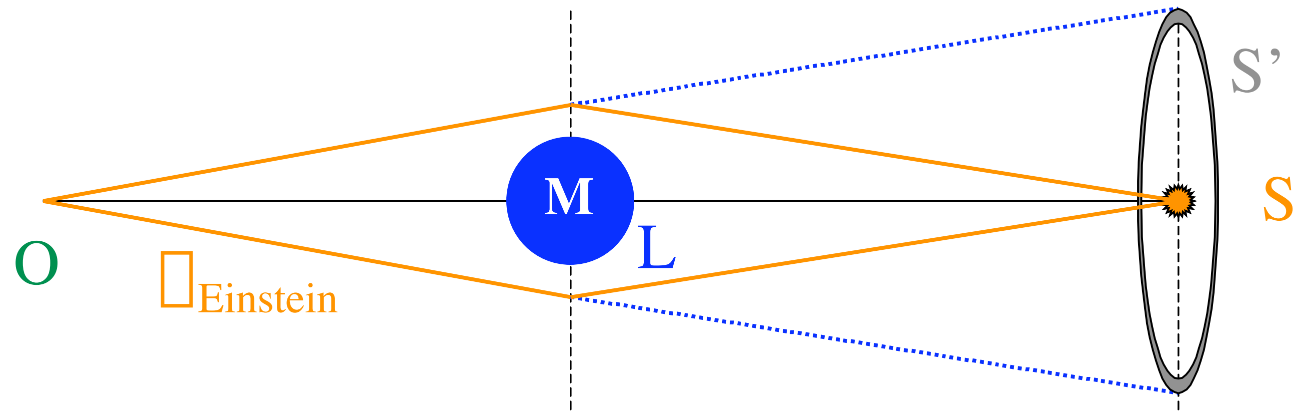
Gravitational Bending of Light



For point mass M deflection angle $\square = \dfrac{4M}{b} \ll 1$

Extended source, integrate. All information in map $\vec{\square}(\vec{\square})$

Strong Lensing and the Einstein Ring

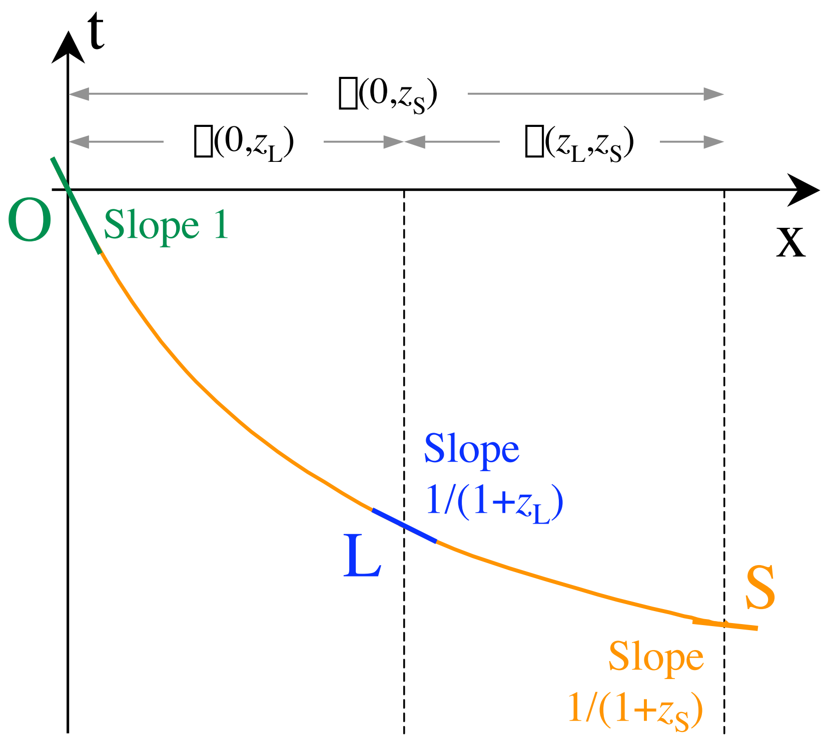


Einstein Radius $\theta_E^2 = \dfrac{4M\,D_{LS}}{D_{OL}D_{OS}}$

For axi-symmetric lens $R < b \Rightarrow \dfrac{M}{\pi R^2} > \dfrac{c^2}{4\pi G}\dfrac{D_{OS}}{D_{OL}D_{LS}} \equiv \Sigma_{Crit}$

For general thin lens $\Sigma > \Sigma_{Crit} \Rightarrow$ multiple images possible

Cosmological Angular Distances



Basic measure $\chi(z_1,z_2)$ = ***comoving distance*** between event of photon emitted at z_2 and observed at z_1

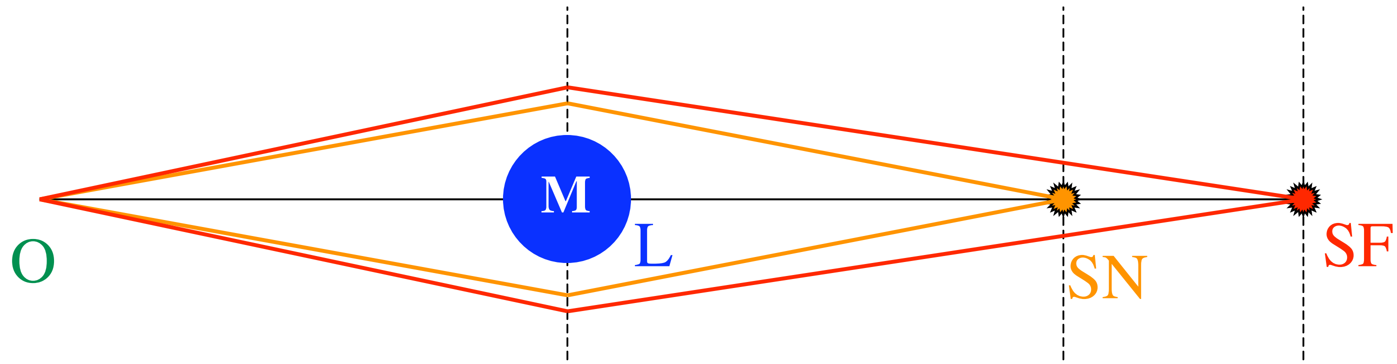
(Bartelmann & Schneider, Phys Rep 340 (2001) 291-472)

Angular diameter distance $D_{ang}(z_1,z_2)=\chi(z_1,z_2)/(1+z_2)$

$$\theta_E^2=\frac{4M\,D_{LS}}{D_{OL}D_{OS}}=\frac{4M(1+z_S)^{-1}\chi(z_L,z_S)}{(1+z_L)^{-1}\chi(0,z_L)(1+z_S)^{-1}\chi(0,z_S)}=4M(1+z_L)\frac{\chi(z_L,z_S)}{\chi(0,z_L)\chi(0,z_S)}$$

$$=4M(1+z_L)\frac{\chi_{LS}}{\chi_{OL}\chi_{OS}}=4M(1+z_L)\frac{\chi_{OS}-\chi_{OL}}{\chi_{OL}\chi_{OS}}=4M(1+z_L)\left[\frac{1}{\chi_{OL}}-\frac{1}{\chi_{OS}}\right]$$

Coincident Double Strong Lensing



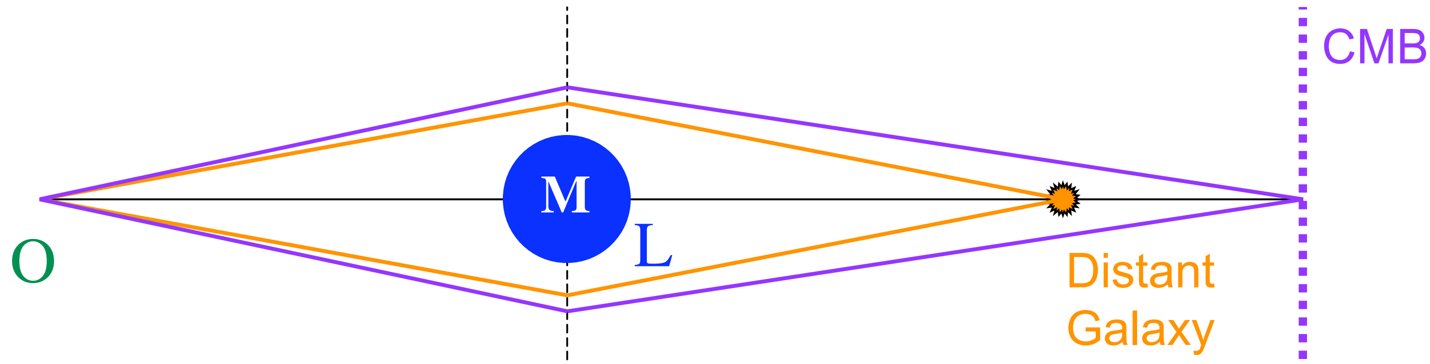
Ratio of areas of near and far Einstein rings:

$$\frac{\theta^2_{\mathrm{EN}}}{\theta^2_{\mathrm{EF}}} = \frac{4M(1+z_L)}{4M(1+z_L)} \quad \frac{1/D_{OL}(z_L) - 1/D_{OSN}(z_{SN})}{1/D_{OL}(z_L) - 1/D_{OSF}(z_{SF})}$$

If we measure $\theta_{\mathrm{EN}}{}^2/\theta_{\mathrm{EF}}{}^2$ and know $D_{\mathrm{OL}}(z_{\mathrm{L}})$ and $D_{\mathrm{OSN}}(z_{\mathrm{SN}})$

then we can map $D_{\mathrm{OSF}}(z_{\mathrm{SF}})$ and measure distance as a function of redshift *without* knowledge of lens mass.

The CMB Shines Behind Everything



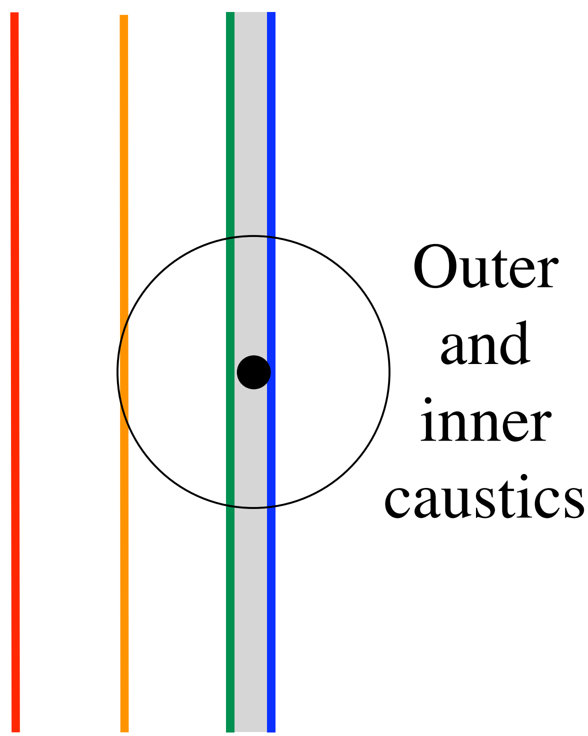
$$\frac{\square^{2}_{\mathrm{EDG}}}{\square^{2}_{\mathrm{ECMB}}} = \frac{1/\square_{OL}(z_L)\,\square\,1/\square_{ODG}(z_{DG})}{1/\square_{OL}(z_L)\,\square\,1/\square_{OCMB}(z_{CMB})}$$

Plan 1: Know $\square_{\mathrm{OL}}(z_{\mathrm{L}})$, $\square_{\mathrm{ODG}}(z_{\mathrm{DG}})$ via Hubble; learn $\square_{\mathrm{OCMB}}$

Plan 2: Know $\square_{\mathrm{OL}}(z_{\mathrm{L}})$, assume $\square_{\mathrm{OCMB}}$ fixed; map $\square_{\mathrm{ODG}}(z_{\mathrm{DG}})$

Einstein Rings in the CMB Image

Expect axi-symmetric strong lens to produce one band (thick isotherm) in the image plane with a unique "□" topology



Source Plane

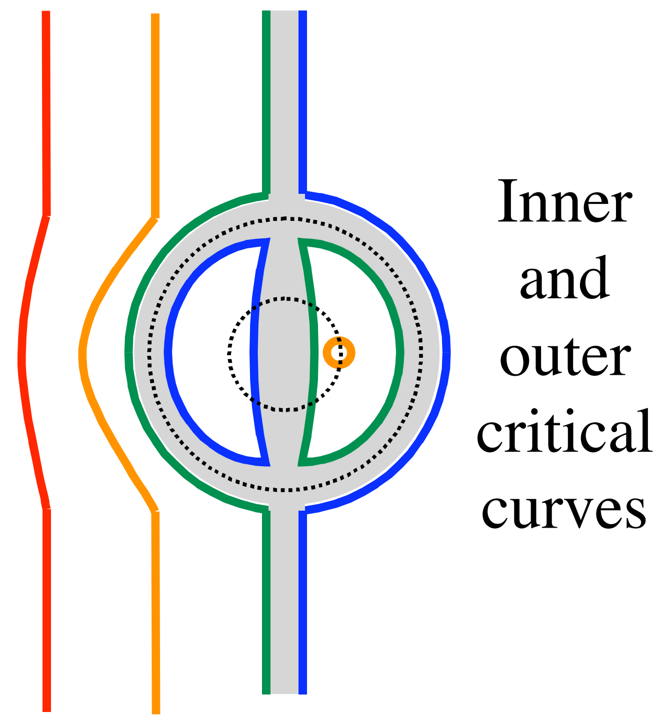


Image Plane

Some work by *actual* experts

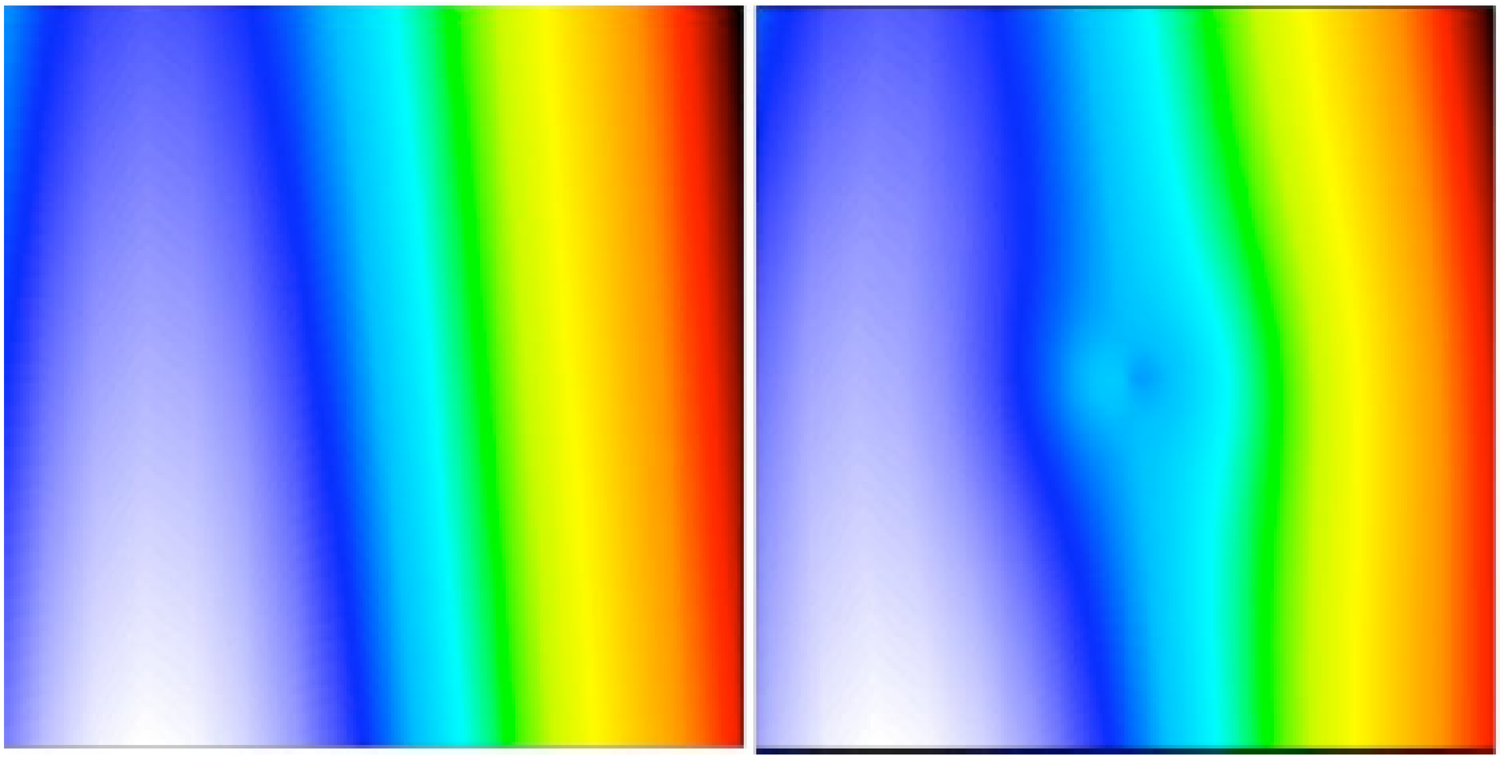


FIG. 3.—*Left panel:* A simulated CMB temperature fluctuation field of 10′ × 10′ size. *Right panel:* The same field, lensed by a galaxy cluster, which imprints a characteristic pattern on the temperature fluctuations.

Simulated CMB temperature field (left) and image after lensing by a galaxy cluster (right) Bartelmann astro-ph/0304162. The “□” feature is clearly visible in the lensed image.

Rude Reality

In extended lenses mass may lie outside Einstein rings

Real lenses are not axi-symmetric; inner caustic no longer degenerate at a point, no "true" Einstein ring

The real CMB is not a pure gradient

Need high spatial resolution (sub-arc-minute) measurement of CMB anisotropies (1 in $10^{5\text{-}6}$ precision)

My conjecture: Even in the real world, strong lensing of the CMB will produce a singular thick isotherm with a unique "□" topology, and the loop area will still correspond to $4□MD_{\mathrm{LCMB}}/D_{\mathrm{OL}}D_{\mathrm{OCMB}}$

(Partial) Reading List

Seljak & Zaldarriaga "Lensing Induced Cluster Signatures in Cosmic Microwave Background" astro-ph/9907254

Dodelson & Starkman, "Galaxy-CMB Lensing", astro-ph/0305467; Dodelson, "CMB-Cluster Lensing", astro-ph/0402314

Maturi et al., "Gravitational lensing of the CMB by galaxy clusters", astro-ph/0408064

Holder & Kosowsky, "Gravitational Lensing of the Microwave Background by Galaxy Clusters", astro-ph/0401519

The subject of GLCMB is taking off! Most work to date has been on using GLCMB to reconstruct mass distributions in clusters. The utility of coincident double strong lensing is relatively under-explored at present.